

Two-dimensional rocking ratchet for cold atoms

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We investigate experimentally a two-dimensional rocking ratchet for cold atoms, realized by using a driven three-beam dissipative optical lattice. AC forces are applied in perpendicular directions by phase-modulating two of the lattice beams. As predicted by the general theory [S. Denisov *et al.*, Phys. Rev. Lett. **100**, 224102 (2008)], we observe a rectification phenomenon unique to high-dimensional rocking ratchets, as determined by two single-harmonic drivings applied in orthogonal directions. Also, by applying two bi-harmonic forces in perpendicular directions, we demonstrate the possibility of generating a current in an arbitrary direction within the optical lattice plane.

PACS numbers:

I. INTRODUCTION

The ratchet effect [1] consists in the rectification of Brownian motion within a macroscopically flat potential, and corresponding generation of a current. Due to the second principle of thermodynamics, directed motion in such a situation can only be obtained in out-of-equilibrium systems. Furthermore, it is necessary to break all the system's symmetries which would otherwise prevent directed motion [2–5]. The ratchet effect is very general, as well exemplified by the variety of systems in which it has been demonstrated: from colloidal particles [6] and solid state devices [7–9] to cold atoms in optical lattices [10–14], synthetic molecules [15] and granular gases [16], just to name a few. Among the different possible implementations of the ratchet effect, one-dimensional rocking ratchets have been studied in great detail. In these ratchets, Brownian particles in an asymmetric periodic potential are driven out of equilibrium by a time-symmetric oscillating force. The particles are set into directed motion due to the asymmetry of the potential landscape. The same effect can be obtained for a spatially symmetric potential and a temporally asymmetric drive [2, 3, 17, 18]. A typical choice for a time asymmetric drive is a bi-harmonic force, with the relative phase between harmonics determining the time-symmetry of the drive [2–5]. Mixing of harmonics with different parity produces then directed motion through the spatially symmetric potential [17].

Recently, the possibility of controlling the motion, via the ratchet effect, within a 2D or a 3D structure has been attracting much attention. In this context, a 3D Brownian motor was demonstrated by using cold atoms in *undriven* optical lattices [14]. Stimulated by recent theoretical work [5, 19, 20], in the present work we consider 2D *ac driven* rocking ratchets for cold atoms as a model system to investigate rectification phenomena unique to these higher dimensional systems, as produced by the interplay between drivings applied in orthogonal directions. We demonstrate that directed motion can be obtained by two single-harmonic drivings applied in orthogonal directions, a rectification phenomenon unique

to high-dimensional rocking ratchets. Furthermore, by applying two bi-harmonic drivings in orthogonal directions, we demonstrate the possibility of generating a current in an arbitrary direction within the optical lattice plane.

II. EXPERIMENTAL SET-UP

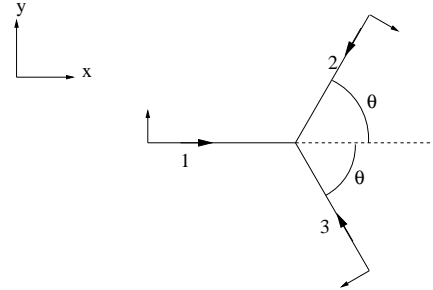


FIG. 1: Lattice beam configuration for the 2D near-resonant optical lattice used in this work. The intensity per lattice beam is $I_L = 70 \text{ mW/cm}^2$. The detuning of the lattice fields from resonance with the D_2 -line $F_g = 2 \rightarrow F_g = 3$ atomic transition is $\Delta = -15\Gamma$. The angle θ is $\theta = 60^\circ$.

Our ratchet set-up consists of a dissipative 2D optical lattice generated by three laser beams of equal intensity [21]. The lattice beam configuration is shown in Fig. 1: three linearly polarized travelling waves propagate in the xy plane; their propagation directions are separated by 120° , and their linear polarizations are all in the xy plane. This beam configuration gives rise to a periodic 2D optical lattice, with the potential minima arranged on a hexagonal lattice. The use of just three beams to generate a 2D lattice has an important advantage [21]: changes in the relative phases between the different laser beams do not produce a change in the topography of the optical potential, but only result in a shift of the lattice as a whole. This allows us to introduce rocking forces in the x - and y - directions by phase-modulating two of the lattice beams. The basic idea is to operate a moving optical lattice, with time-dependent acceleration \mathbf{a} .

Then, in the non-inertial reference frame of the lattice, an inertial force $\mathbf{F} = -m\mathbf{a}$ is introduced, where m is the atomic mass. In the specific case considered here, if we modulate beams 2 and 3, and indicate the resulting time-dependent phases by ϕ_2 and ϕ_3 respectively, we find that in the reference frame of the lattice an inertial force \mathbf{F} appears with components equal to:

$$\mathbf{F} = (F_x, F_y) = -m(a_x, a_y) = -\frac{m}{k}(\frac{\ddot{s}}{3}, \frac{\ddot{\delta}}{\sqrt{3}}) \quad (1)$$

Here a_x , a_y are the components of the lattice acceleration, k the laser beam's wavevector, s and δ are the sum and the difference of the phases of beams 1 and 2: $s = \phi_2 + \phi_3$, $\delta = \phi_3 - \phi_2$. In the experiment, the three lattice beams are obtained from the same laser beam. This laser beam is split into three beams, which are passed through three acousto-optical modulators (AOM1, AOM2 and AOM3 for the beams 1,2 and 3, respectively) driven by three RF generators at 80 MHz. The first-order diffracted beams are then used to generate the optical lattice. Three additional phase-locked frequency generators are used to produce the rocking forces F_x and F_y . One generator produces the signals $V_1 = A_x \sin(\omega t)$ and $V_2 = A_y \sin(\omega t)$, while the other two generators produce $V_3 = B_x \sin(2\omega t + \phi_x)$ and $V_4 = B_y \sin(2\omega t + \phi_y)$ with $\omega = 2\pi \cdot 50$ kHz for all experiments reported in this work. We then add the signals two by two, so to obtain:

$$V_x = A_x \sin(\omega t) + B_x \sin(2\omega t + \phi_x) \quad (2a)$$

$$V_y = A_y \sin(\omega t) + B_y \sin(2\omega t + \phi_y) . \quad (2b)$$

The sum $V_x + V_y$ is then used as frequency-modulation input signal for the RF generator driving AOM2, so to phase modulate lattice beam 2. In the same way, the difference $V_x - V_y$ is used to phase-modulate, via the RF generator driving AOM3, the lattice beam 3. According to Eq. 1 this produces the following ac forces:

$$F_x = -\frac{2m\omega f_0}{3k} [A_x \cos(\omega t) + 2B_x \cos(2\omega t + \phi_x)] \quad (3a)$$

$$F_y = -\frac{2m\omega f_0}{\sqrt{3}k} [A_y \cos(\omega t) + 2B_y \cos(2\omega t + \phi_y)] \quad (3b)$$

where $f_0 = 500$ kHz, as determined by the frequency modulation depth of the RF generators.

III. SYMMETRY ANALYSIS

Before presenting the findings of our experiments, we recall the essential elements of the symmetry analysis for a two-dimensional rocking ratchet [5]. Such an analysis will allow us to interpret the experimental results. Consider a particle moving in a bi-dimensional potential $V(x, y)$ and driven by a periodic zero-average ac force $\mathbf{F}(t)$ of period T . The relevant symmetries for the directed transport through the potential are those which

reverse the sign of the momentum:

$$S_1 : \quad \mathbf{r} \rightarrow -\mathbf{r} + \mathbf{r}', \quad t \rightarrow t + \tau \quad (4)$$

$$S_2 : \quad \mathbf{r} \rightarrow \mathbf{r} + \mathbf{\Lambda}, \quad t \rightarrow -t + t', \quad (5)$$

where \mathbf{r}' , τ , $\mathbf{\Lambda}$ and t' are constants which represent shifts in time and space. If the system is invariant under S_1 and/or S_2 , directed transport is forbidden. Whether S_1 , S_2 are symmetries of the system depends on the symmetry properties of the potential and the driving. If the potential is symmetric, i.e. $V(-\mathbf{r} + \mathbf{r}') = V(\mathbf{r})$, and the driving is *shift-symmetric*, i.e. $\mathbf{F}(t + T/2) = -\mathbf{F}(t)$, then S_1 is a symmetry of the system. Moreover, in the Hamiltonian limit S_2 holds if the driving is symmetric under time-reversal, i.e. $\mathbf{F}(-t + t') = \mathbf{F}(t)$.

In the following, the effect of the application of different ac forces will be studied by analyzing the breaking of the relevant symmetries.

IV. EXPERIMENTAL RESULTS

The experimental procedure is the following. We cool and trap ^{87}Rb atoms in a magneto-optical trap (MOT). Once the trap is loaded, we turn off the MOT beams and magnetic field, and load the atoms in the aforescribed 2D optical lattice. The ac drivings are then turned on by increasing linearly with time their amplitudes from zero to the wanted value in 1 ms. The motion of the atoms in the driven lattice is then studied with the help of a CCD camera using absorption imaging. This allows us to determine the position of the atomic cloud center of mass at a given instant.

A. 1D ratchet effect

In the first set of measurements, we apply a bi-harmonic driving along the x - or the y -direction, and monitor the atomic center-of-mass (CM) motion. Results for the x - and y - components of the CM velocity are reported in Fig. 2 as a function of the phase difference between the driving harmonics. The data of Fig. 2(a,b) refer to a bi-harmonic driving along the x -direction, while the data of Fig. 2(c,d) refer to a driving along the y direction. These measurements show that a bi-harmonic driving in the x - (respectively, y -) direction leads to a ratchet effect only in that direction, with the current generated showing a sinusoidal dependence on the phase between driving harmonics.

We notice that for a bi-harmonic driving applied in one direction only, the symmetry analysis reduces to the one for a 1D rocking ratchet [2, 11, 12]. This is consistent with our results of Fig. 2, with a current generated in the direction of the driving only.

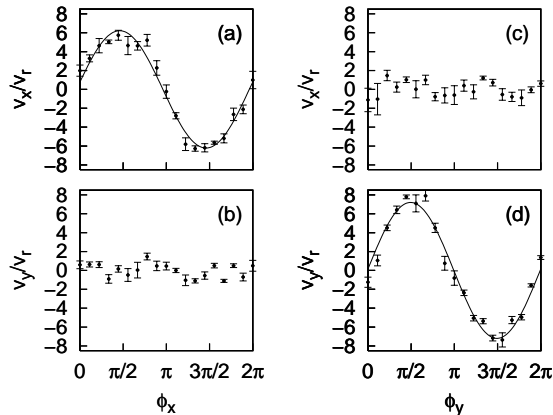


FIG. 2: Experimental results for the x - and y - components of the center-of-mass velocity of the atomic cloud as a function of the phase between the driving harmonics. The left column (plots (a) and (b)) refers to a bi-harmonic driving along x . The right column (plots (c) and (d)) refers to a bi-harmonic driving along y . The parameters of the driving (see Eq. 3) are: $A_x = 1.5\text{V}$, $B_x = 2.0\text{V}$, $A_y = B_y = 0\text{V}$ for (a) and (b), and $A_x = B_x = 0\text{V}$, $A_y = 1.5\text{V}$, $B_y = 2.0\text{V}$ for (c) and (d). The solid lines are the best fit of the data with the function $v_{x,y} = v_{\max}^{x,y} \sin(\phi_{x,y} + \phi_0^{x,y})$.

B. Split bi-harmonic driving

Consider now the case of simultaneous driving along the x - and y - directions. We examine first the case of a single harmonic driving, of frequency ω , along the x -direction and an additional single harmonic driving, with frequency 2ω and relative phase ϕ , along y ("split bi-harmonic driving"). The results of our measurements for this case are reported in Fig. 3. For this configuration a phase-dependent current is generated along the y direction, while no current is generated, within the experimental error, in the x direction. Such a generation of a current can be understood in the framework of the symmetry analysis [5]. We consider the Hamiltonian limit, with a phase-shift accounting for weak dissipation. This is appropriate for a weakly-dissipative rocking ratchet, as the ones realized by using near-resonant optical lattices [12, 22]. The driving is of the form

$$\mathbf{F} = \hat{\mathbf{x}}F_x^\circ \cos(\omega t) + \hat{\mathbf{y}}F_y^\circ \cos(2\omega t + \phi), \quad (6)$$

with the period of the driving equal to $T = 2\pi/\omega$. As we have $V(x, y) = V(-x, y)$ and $F_x(t + T/2) = -F_x(t)$, directed transport along x is forbidden by symmetry. On the other hand, the y -component of the driving force is not shift-symmetric as $F_y(t + T/2) \neq -F_y(t)$. Thus, transport along y is not forbidden by symmetry. Such a transport is then controlled by S_2 , which is realized for $\phi = n\pi$, with n integer. The above symmetry analysis explains the observed generation of a phase-dependent sine-like current along y , while no current is generated

along x . We obtained analogous results for the reversed situation: by applying a single harmonic driving along y with frequency ω , and a single harmonic driving along x with frequency 2ω and relative phase ϕ , we observed the generation of a phase-dependent sine-like current along the x -direction.

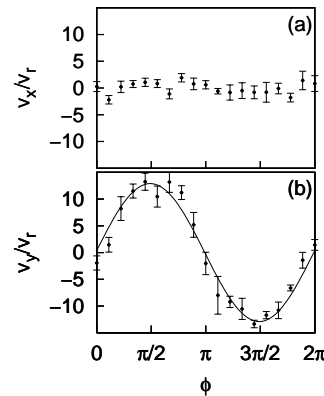


FIG. 3: Experimental results for the x - and y - components of the center-of-mass velocity of the atomic cloud as a function of the phase between the driving harmonics. The driving consists of a single harmonic driving, with frequency ω , along x and an additional single harmonic driving, with frequency 2ω and relative phase ϕ , along y . The parameters of the driving (see Eq.3) are: $A_x = 1.5\text{V}$, $B_x = 0\text{V}$, $A_y = 0\text{V}$, $B_y = 2.0\text{V}$. The solid line is the best fit of the data with the function $v_y = v_{\max} \sin(\phi + \phi_0)$.

These results constitute a first experimental demonstration of a rectification phenomenon unique to high dimensional rocking ratchets, as predicted by the symmetry analysis of Denisov *et al* [5]. Furthermore, these results show that for the considered lattice geometry the x - and y -directions are coupled under the influence of ac drivings. That is, the application of two single-harmonic drivings along the x - and y - directions, which taken one-by-one would not break the relevant symmetries and would therefore not induce any directed motion, can break the symmetries of the system and lead to directed motion. Under the action of these forces the atoms explore orbits in 2D [5] while generating an average drift in the direction of the driving at the frequency 2ω .

C. Control of directed motion in 2D

We now examine the implementation of a 2D rocking ratchet in which directed motion can be controlled in the xy plane. We consider the case of two bi-harmonic drivings applied simultaneously, one in the x - and one in the y -direction. The results presented so far - see Fig. 2 and related discussion - showed that by applying a single bi-harmonic driving along the x or y direction we can

generate a current in the direction of the driving. The issue addressed now is whether by applying simultaneously two bi-harmonic drivings, in the x and y directions respectively, it is possible to generate a current in an arbitrary direction in the xy plane.

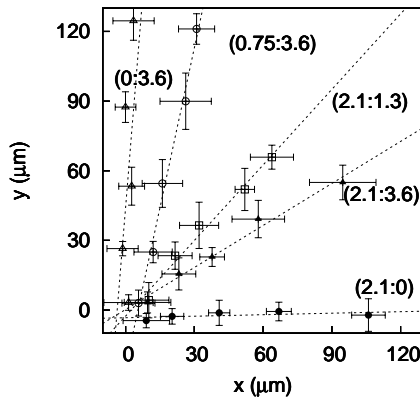


FIG. 4: Position of the atomic cloud center-of-mass at different instants. The different data sets correspond to different relative amplitudes of F_x and F_y , and are labelled by $(V_x^{pp}/\sqrt{3} : V_y^{pp})$, where V_x^{pp} (respectively, V_y^{pp}) is the peak-to-peak amplitude of the signal - see Eq. 2 - leading to the generation of the ac force in the x (respectively, y) direction. As from Eq. 3, the ratio $(V_x^{pp}/\sqrt{3})/V_y^{pp}$ is proportional to the forces amplitude ratio F_x/F_y . For each data set, the point closer to the origin corresponds to the first image, taken immediately after the ac drivings were ramped up. The other points correspond to images taken at intervals of 0.5 ms. The lines are the best fit of the data with a linear function.

We consider two-biharmonic driving forces F_x , F_y of the form specified by Eq. 3. We take the two forces to have the same temporal dependence by choosing the same relative phases between harmonics: $\phi_x = \phi_y = \pi/2$ and the same ratio between harmonic amplitudes: $A_x/B_x = A_y/B_y = 3/4$. We notice that the choice of the values for the phases is such to break all the relevant symmetries which would otherwise inhibit the generation of a current. This is confirmed by our results of Fig. 2: for this choice of phases and ratio between harmonic amplitudes, F_x and F_y individually can generate directed motion along x and y respectively.

In the experiment, we apply F_x and F_y simultaneously and monitor the resulting motion of the atomic center-of-mass. The results of our measurements are reported in Fig. 4, where the center-of-mass of the atomic-cloud is plotted at different instants. The different series of measurements correspond to different relative weights of F_x and F_y , as obtained by varying the overall amplitudes

of F_x and F_y while keeping their temporal dependence unchanged. Our measurements show that the atoms are set into directed motion as a result of the combined action of the drivings in the x - and y - directions. And it is possible, by choosing appropriately the relative weights of the drivings in the x and y directions, to generate a current in a wanted direction.

It is interesting to stress some characteristics of the generation of a current in a 2D rocking ratchet, as from Fig. 4. The generated current should not be interpreted as consisting of a x -component v_x generated by F_x only and by a y -component v_y generated by F_y only. This because there is a cross-coupling between orthogonal directions, as best exemplified by our results of a generation of a current following two single-harmonic drivings in the x - and y -directions respectively - see Fig. 3. This clearly produces a complicated dependence of the directions of the obtained current on the amplitudes of F_x and F_y . Moreover, we notice that even for a 1D ratchet the current amplitude is a non-monotonic function of the driving amplitude [11]. Therefore, even neglecting cross-couplings, we do not expect the direction of the current to vary monotonically with the ratio of the amplitudes between the bi-harmonic drivings in the x - and y -directions.

V. CONCLUSIONS

In conclusion, we investigated a two-dimensional rocking ratchet for cold atoms realized using ac driven dissipative optical lattices. In these optical lattices the excess energy introduced by the driving is dissipated by the friction force associated to the Sisyphus cooling mechanism [21], and it is removed from the system by the scattered photons. We observed a rectification phenomenon unique to high-dimensional rocking ratchets, as determined by two single-harmonic drivings applied in orthogonal directions, and demonstrated the possibility of generating a current in an arbitrary direction within the optical lattice plane.

The set-up demonstrated in this work will also allow one to investigate several other phenomena specific to high-dimensional rocking ratchets, as the generation and control of vorticity [5] or the transverse rectification of fluctuations resulting from orthogonally applied dc and ac drivings [20]. The results presented in this work are also of relevance to fluxtronic devices, in which the motion of flux quanta within a 2D spatially symmetric landscape is controlled by time-asymmetric ac fields [23].

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